Hanoi Open Mathematics Competition
Team Contest - Junior Section
Time limit: 60 minutes
Sample Questions

Information:

- You are allowed 60 minutes to complete 10 questions. For Questions 1, 2, 3, 4, 5, 6, 7, and 8, only numerical answers are required. For Questions 9 and 10, full solutions are required.
- Each one of Questions 1, 2, 3, and 4 is worth 5 points, and each one of Questions 5, 6, 7, and 8 is worth 10 points. No partial credits are given, and there are no penalties for incorrect answers. Each one of Questions 9 and 10 is worth 20 points, and partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on the first page.
- Enter your answers in the space provided below the question.
- All together may discuss and complete the questions.
- The instruments such as protractors, calculators and electronic devices are not allowed to use.
- At the end of the contest you must put the question papers in the envelope provided.
- Write down your team's name in the space provided on every question sheet.

Team: ______________________ **Score:** ______________

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No.	Questions										Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10		
Score												

Question 1. The terms in the following sequence are all fractions:

$$\frac{1}{2},\frac{1}{3},\frac{2}{4},\frac{3}{5},\frac{1}{6},\frac{2}{7},\frac{3}{8},\frac{4}{9},\frac{5}{2},\frac{1}{3},\frac{2}{4},\frac{3}{5},\frac{4}{6},\frac{5}{7},\frac{6}{8},\frac{7}{9},\frac{1}{2},\ldots$$

Find the 9999th term of the sequence.

Answer: ______________________________

Question 2: Let ABC be a right triangle at A such that $AB=3, AC=4$. Let E, F be two points on the sides AB and AC, respectively such that $\angle AEF=\angle ACB$ and $\angle AFE=\angle ABC$. The perpendiculars drew from E,F to BC meet BC at points P and Q, respectively. Calculate $S=PE+EF+FQ$.

Answer: ______________________________

Question 3: Let x,y,z be real numbers such that

$$\begin{cases} x^3+y=x^2+2 \\ 2y^3+z=4y^2+3 \\ 3z^3+x=9z^2+1 \end{cases}$$

Evaluate $P=xyz$.

Answer: ______________________________

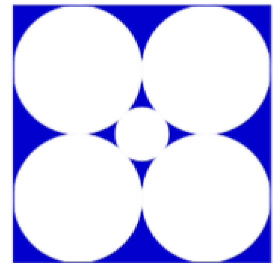
Question 4: There are 2017 points inside a convex polygon of 2017 sides whose area is equal to 1. Assume that arbitrary three points of the 4034 given points are not collinear, and there exists a triangle with three vertices taken from 4034 given points whose area does not exceed x. Find x.

Answer: ______________________________

Question 5. Calculate the sum of all natural numbers n such that $(n+1)!-n+29$ is divisible by $n!+n+1$.

Answer: ______________________________

Question 6. As shown in the figure, the square alongside has sides of length 4 units. The four identical circles fit tightly inside the square and the small circle that will fit in the central hole. What is the area of the shaded?



Answer: ______________________________

Question 7. Find the 3-digit number $\overline{abc}$ such that $\overline{abc}+\overline{bca}+\overline{bac}+\overline{cab}+\overline{cba}=3194$.

Answer: ______________________________

Question 8. Solve the equation

$$\left(\frac{x}{x-1}\right)^2+\left(\frac{x}{x+1}\right)^2=90$$

Answer: ______________________________

Question 9. Let a,b,c be given distinct real numbers. Solve the equation :

$$\frac{(b-c)(1+a^2)}{x+a^2}+\frac{(c-a)(1+b^2)}{x+b^2}+\frac{(a-b)(1+c^2)}{x+c^2}=0.$$

Solution:

Answer: ______________________________

Question 10. Given $\Delta ABC\left(AB=c;AC=b;BC=a\right)$ with its incenter I. Prove that

$$\frac{IA^2}{bc}+\frac{IB^2}{ca}+\frac{IC^2}{ab}=1\,.$$

Solution:

Answer: ______________________________