Hanoi Open Mathematics Competition
Individual Contest - Senior Section
Time limit: 120 minutes
Sample Questions

Information:

- You are allowed 120 minutes to complete 10 questions in Section A to which only numerical answers are required, and 5 questions in Section B to which full solutions are required.
- Each one of Questions1, 2, 3, 4, and 5 is worth 5 points, and each one of Questions 6, 7, 8, 9, and 10 is worth 10 points. No partial credits are given, and there are no penalties for incorrect answers. Each one of Question 11, 12, 13, 14, and 15 is worth 15 points, and partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- For Section A, enter your short answers in the provided space. For Section B, write down your full solutions.
- You must use either pencil or ball-point pen which is either black or blue.
- The instruments such as protractors, calculators and electronic devices are not allowed to use.

Team: ________________ **Name:** ___________________________ **No.:**_______ **Score:** _______

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No.	Section A										Section B					Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
Score																	

Section A.

In this section, there are 10 questions. Fill in your answer in the space provided at the end of each question.

Question 1: Let $x = 2+\sqrt{3}$, Then $x^4 + \frac{1}{x^4}$. is

A. 196. **B.** 194. **C.** $14+2\sqrt{3}$. **D.** $14-2\sqrt{3}$. **E.** $190\sqrt{3}$.

Question 2: Given a circle with center O and diameter AB of length 2. The perpendicular from the midpoint Q of OA intersects the circle at P. The radius of the circle which can be inscribed in triangle APB is

A. $\frac{\sqrt{3}+1}{2}$ **B.** $\frac{\sqrt{3}-1}{2}$ **C.** $\frac{2-\sqrt{3}}{4}$ **D.** $\frac{2+\sqrt{3}}{4}$. **E.** $\frac{\sqrt{3}}{2}$.

Question 3: Let $x = \frac{\left(\sqrt{3}-1\right)\sqrt[3]{10+6\sqrt{3}}}{\sqrt{6+2\sqrt{5}}-\sqrt{5}}$. Evaluate $P = \left(x^3 - 4x + 1\right)^{2018}$

A. $2\sqrt{5}$. **B.** 18. **C.** 1 **D.** 2. **E.** $3\sqrt{2}$.

Question 4: Let $P(x)$ be a monic polynomial of degree 3. (*Monic* here means that the coefficient of x^3 is 1). Suppose that the remainder when $P(x)$ is divided by $x^2 - 5x + 6$ equals 2 times the remainder when $P(x)$ is divided by $x^2 - 5x + 4$. If $P(0) = 100$, what is $P(5)$?

A. 112. **B.** 110. **C.** 108. **D.** 106. **E.** 104.

Question 5: Let a, b, and c be distinct nonzero real numbers with

$$\frac{1+a^3}{a} = \frac{1+b^3}{b} = \frac{1+c^3}{c}.$$

Find the value of $S = a^3 + b^3 + c^3$.

A. $S = -3$. **B.** $S = -6$. **C.** $S = -9$. **D.** $S = 3$. **E.** $S = 6$.

Question 6: Solve the equation $\left(4x-1\right)\sqrt{x^2+1} = 2x^2 + 2x + 1$.

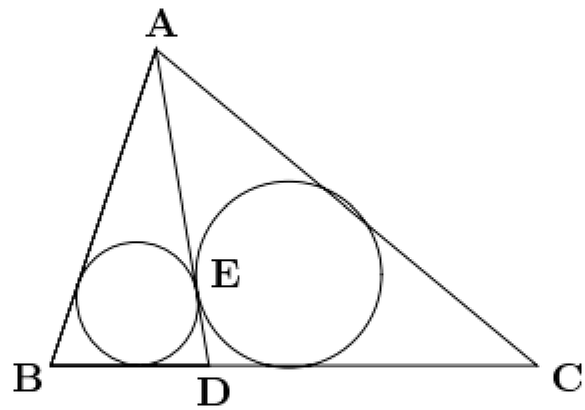
Answer: ____________________

Question 7: Given positive real numbers x, y, z with $x - \sqrt{y+6} = \sqrt{x+6} - y$. Assume that the minimum and maximum value of $S = x + y$ are M and m respectively. Determine

$$A = \frac{M+m}{2}.$$

Answer: ____________________

Question 8: The triangle ABC has sides AB = 137; AC = 241, and BC = 200. There is a point D on BC such that both incircles of triangles ABD and ACD touch AD at the same point E. Determine the length of CD.



Answer: ____________________

Question 9: Find all real values of x; y and z such that

$$\begin{cases} x - \sqrt{yz} = 42 \\ y - \sqrt{zx} = 6 \\ z - \sqrt{xy} = -30 \end{cases}.$$

Answer: ____________________

Question 10: A rectangular box P is inscribed in a sphere of radius r. The surface area of P is 384, and the sum of the lengths of its 12 edges is 112. What is r?

Answer: ____________________

Section B.

Answer the following 5 questions. Show your detailed solution in the space provided.

Question 11: Let AB and CD be two mutually perpendicular chords of a circle with radius R, and let I be the intersection of AB and CD. Prove that $IA^2 + IB^2 + IC^2 + ID^2 = 4R^2$.

Solution:

Answer: ____________________

Question 12: Solve in positive integers: $520\left(xyzt+xy+xz+zt+1\right)=577\left(yzt+y+z\right)$.

Solution:

Answer: ____________________

Question 13: Prove that the number

$$\frac{\left(2^4+\frac{1}{4}\right)\left(4^4+\frac{1}{4}\right)\left(6^4+\frac{1}{4}\right)\left(8^4+\frac{1}{4}\right)\left(10^4+\frac{1}{4}\right)\left(12^4+\frac{1}{4}\right)}{\left(1^4+\frac{1}{4}\right)\left(3^4+\frac{1}{4}\right)\left(5^4+\frac{1}{4}\right)\left(7^4+\frac{1}{4}\right)\left(9^4+\frac{1}{4}\right)\left(11^4+\frac{1}{4}\right)}$$

is an integer, and find the number by simplification without actual calculations.

Solution:

Answer: ____________________

Question 14: Let (O, R_1) and (O, R_2) be two concentric circles with radii $R_1<R_2$. Let l and m be two parallel chords of (O, R_2) which are tangent to the inner circle (O, R_1), and let A be a point on the outer circle (O, R_2) but it is inside the strip of l and m. The tangents to the inner circle through A with their intersection points C and D with the chords. Prove that the product AC$\times$AD does not depend on the position of A.

Solution:

Answer: ____________________

Question 15: Given real numbers x, y, z with

$$\begin{cases} a^2 + b^2 + 2a - 4b + 4 = 0 \\ c^2 + d^2 - 4c + 4d + 4 = 0 \end{cases}.$$

Find the minimum and maximum of $P = (a-c)^2 + (b-d)^2$.

Solution:

Answer: ____________________